What is claimed is:

1. A method of model reduction and system identification of a dynamic system with multiple inputs, comprising:

generating a plurality of statistically independent random numbers for use as input signals; and

performing a singular-value-decomposition directly on a system response of the dynamic system due to a simultaneous excitation of the plurality of input signals.

2. The method of Claim 1, further comprising sampling individual pulse responses for a first time step and a second time step.

3. The method of Claim 1, further comprising sampling a system response y^n for $n = 0, 1, 2, \ldots M$.

4. The method of Claim 1, further comprising defining Hankel-like matrices $\mathbf{H}_{c0}$ and $\mathbf{H}_{c1}$ as follows:

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$$
\begin{aligned}
\mathbf{H}_{c0} &\equiv [\mathbf{y}_{c0}^{1}\ \mathbf{y}_{c0}^{2}\ \cdots\ \mathbf{y}_{c0}^{M-1}] \\
&= \mathbf{C}[\mathbf{x}^{1}\ \mathbf{x}^{2}\ \ldots\ \mathbf{x}^{M-1}] \qquad (25)\\
\mathbf{H}_{c1} &\equiv [\mathbf{y}_{c1}^{1}\ \mathbf{y}_{c1}^{2}\ \cdots\ \mathbf{y}_{c1}^{M-1}] \\
&= \mathbf{CA}[\mathbf{x}^{1}\ \mathbf{x}^{2}\ \ldots\ \mathbf{x}^{M-1}] \qquad (26)
\end{aligned}
$$

SVD of $\mathbf{H}_{c0}$ yields

$$
\begin{aligned}
\mathbf{H}_{c0} &\equiv \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{T} \\
&\simeq [\mathbf{U}_{R}\ \mathbf{U}_{D}]\begin{bmatrix}\boldsymbol{\Sigma}_{R} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{V}_{R}^{T}\\ \mathbf{V}_{D}^{T}\end{bmatrix} \\
&= \mathbf{U}_{R}\boldsymbol{\Sigma}_{R}^{1/2}\boldsymbol{\Sigma}_{R}^{1/2}\mathbf{V}_{R}^{T} \qquad (27)
\end{aligned}
$$

5. The method of Claim 4, further comprising obtaining system matrices **(A, B, C, D)** by a least square approximation as follows:

$$
\begin{aligned}
\mathbf{D} &= \mathbf{Y}^{0} && (28)\\
\mathbf{C} &\simeq \mathbf{U}_{R}\boldsymbol{\Sigma}_{R}^{1/2} && (29)\\
\mathbf{B} &\simeq \boldsymbol{\Sigma}_{R}^{-1/2}\mathbf{U}_{R}^{T}\mathbf{Y}^{1} && (30)\\
\mathbf{A} &\simeq \boldsymbol{\Sigma}_{R}^{-1/2}\mathbf{U}_{R}^{T}\mathbf{H}_{c1}\mathbf{V}_{R}\boldsymbol{\Sigma}_{R}^{-1/2} && (31)
\end{aligned}
$$

6. The method of Claim 4, wherein $(M - 1) \geq R$ and $N_0 \geq R$.

7. The method of Claim 4, wherein a total number of input samples is equal to $M + 1 + 2 x N_i$.

8. The method of Claim 1, further comprising defining augmented $\mathbf{H}_{c01}$ and $\mathbf{H}_{c11}$ matrices as follows:

$$
\begin{aligned}
\mathbf{H}_{c01} &\equiv \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \\ \vdots \\ \mathbf{CA}^K \end{bmatrix} [\mathbf{x}^1 \ \mathbf{x}^2 \ \dots \ \mathbf{x}^{M-1}] \\
&= \begin{bmatrix} y_{c0}^1 & y_{c0}^2 & \cdots & y_{c0}^{M-1} \\ y_{c1}^1 & y_{c1}^2 & \cdots & y_{c1}^{M-1} \\ \cdots & \cdots & \cdots & \cdots \\ y_{cK}^1 & y_{cK}^2 & \cdots & y_{cK}^{M-1} \end{bmatrix} \qquad (32) \\
\mathbf{H}_{c11} &\equiv \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \\ \vdots \\ \mathbf{CA}^K \end{bmatrix} \mathbf{A}[\mathbf{x}^1 \ \mathbf{x}^2 \ \dots \ \mathbf{x}^{M-1}] \\
&= \begin{bmatrix} y_{c1}^1 & y_{c1}^2 & \cdots & y_{c1}^{M-1} \\ y_{c2}^1 & y_{c2}^2 & \cdots & y_{c2}^{M-1} \\ \cdots & \cdots & \cdots & \cdots \\ y_{cK+1}^1 & y_{cK+1}^2 & \cdots & y_{cK+1}^{M-1} \end{bmatrix} \qquad (33)
\end{aligned}
$$

where

$$
\begin{aligned}
y_{ck}^n &\equiv \mathbf{CA}^k \mathbf{x}^n \\
&= y^{n+k} - \sum_{i=1}^{N_i} y_i^0 r_i^{n+k} - \sum_{i=1}^{N_i} y_i^1 r_i^{n+k-1} - \\
&\quad \dots - \sum_{i=1}^{N_i} y_i^k r_i^n
\end{aligned}
$$

9. The method of Claim 8, wherein a total number of input samples is equal to $M+1+K+(2+K) \times N_i$.

10. The method of Claim 1, wherein at least some of the input signals are filtered through a low-pass filter.

11. The method of Claim 1, wherein the plurality of input signals includes applying multiple step inputs in a sequential manner.

12. The method of Claim 1, wherein the plurality of input signals includes applying multiple pulse inputs in a sequential manner.

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13. The method of Claim 1, further comprising performing a second order reduction based on the Frequency-Domain Karhunen-Loeve (FDKL) method to the SCI/ERA ROM using the plurality of input signals.

14. The method of Claim 13, further comprising premultiplying the SCI/ERA ROM matrices by $\mathbf{\Phi}^T$ to yield a new reduced-order model as follows:

$$\mathbf{p}^{n+1} = \mathbf{A}_1 \mathbf{p}^n + \mathbf{B}_1 \mathbf{u}^n \quad (53)$$

$$\mathbf{y}^n = \mathbf{C}_1 \mathbf{p}^n + \mathbf{D} \mathbf{u}^n \quad (54)$$

where

$$\mathbf{A}_1 \equiv \mathbf{\Phi}^T \mathbf{A} \mathbf{\Phi} \quad (55)$$

$$\mathbf{B}_1 \equiv \mathbf{\Phi}^T \mathbf{B} \quad (56)$$

$$\mathbf{C}_1 \equiv \mathbf{C} \mathbf{\Phi} \quad (57)$$

15. A method of model reduction and system identification of a dynamic system with multiple inputs, comprising:

generating a plurality of statistically independent random numbers for use as input signals; and

performing a singular-value-decomposition directly on a system response of the dynamic system due to a simultaneous excitation of the plurality of input signals;

sampling individual pulse responses for a first time step and a second time step;

defining $\mathbf{H}_{c0}$ and $\mathbf{H}_{c1}$ matrices as follows:

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$$
\begin{aligned}
\mathbf{H}_{c0} &\equiv [\mathbf{y}_{c0}^1 \ \mathbf{y}_{c0}^2 \cdots \mathbf{y}_{c0}^{M-1}] \\
&= \mathbf{C}[\mathbf{x}^1 \ \mathbf{x}^2 \ \ldots \ \mathbf{x}^{M-1}] && (25) \\
\mathbf{H}_{c1} &\equiv [\mathbf{y}_{c1}^1 \ \mathbf{y}_{c1}^2 \cdots \mathbf{y}_{c1}^{M-1}] \\
&= \mathbf{C}\mathbf{A}[\mathbf{x}^1 \ \mathbf{x}^2 \ \ldots \ \mathbf{x}^{M-1}] && (26)
\end{aligned}
$$

SVD of $\mathbf{H}_{c0}$ yields

$$
\begin{aligned}
\mathbf{H}_{c0} &\equiv \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^T \\
&\simeq [\mathbf{U}_R \ \mathbf{U}_D] \begin{bmatrix} \boldsymbol{\Sigma}_R & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{V}_R^T \\ \mathbf{V}_D^T \end{bmatrix} \\
&= \mathbf{U}_R \boldsymbol{\Sigma}_R^{1/2} \boldsymbol{\Sigma}_R^{1/2} \mathbf{V}_R^T && (27)
\end{aligned}
$$

; and

obtaining system matrices (**A, B, C, D**) by a least square approximation as follows:

$$
\begin{aligned}
\mathbf{D} &= \mathbf{Y}^0 && (28) \\
\mathbf{C} &\simeq \mathbf{U}_R \boldsymbol{\Sigma}_R^{1/2} && (29) \\
\mathbf{B} &\simeq \boldsymbol{\Sigma}_R^{-1/2} \mathbf{U}_R^T \mathbf{Y}^1 && (30) \\
\mathbf{A} &\simeq \boldsymbol{\Sigma}_R^{-1/2} \mathbf{U}_R^T \mathbf{H}_{c1} \mathbf{V}_R \boldsymbol{\Sigma}_R^{-1/2} && (31)
\end{aligned}
$$

16. The method of Claim 15, wherein at least some of the input signals are filtered through a low-pass filter.

17. The method of Claim 15, wherein the plurality of input signals includes applying multiple step inputs in a sequential manner.

18. The method of Claim 15, wherein the plurality of input signals includes applying multiple pulse inputs in a sequential manner.

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19. The method of Claim 15, further comprising performing a second order reduction using the Frequency-Domain Karhunen-Loeve (FDKL) method to the SCI/ERA ROM using the plurality of input signals.

20. A method of simulating a fluid flow, comprising:

generating a plurality of statistically independent random numbers for use as input signals; and

performing a singular-value-decomposition directly on a fluid response due to a simultaneous excitation of the plurality of input signals.

21. The method of Claim 20, further comprising sampling individual pulse responses for first and second time steps.

22. The method of Claim 20, further comprising defining $\mathbf{H}_{c0}$ and $\mathbf{H}_{c1}$ matrices as follows:

$$\begin{aligned} \mathbf{H}_{c0} &\equiv [\mathbf{y}_{c0}^1 \ \mathbf{y}_{c0}^2 \cdots \mathbf{y}_{c0}^{M-1}] \\ &= \mathbf{C}[\mathbf{x}^1 \ \mathbf{x}^2 \ \ldots \ \mathbf{x}^{M-1}] \end{aligned} \quad (25)$$

$$\begin{aligned} \mathbf{H}_{c1} &\equiv [\mathbf{y}_{c1}^1 \ \mathbf{y}_{c1}^2 \cdots \mathbf{y}_{c1}^{M-1}] \\ &= \mathbf{CA}[\mathbf{x}^1 \ \mathbf{x}^2 \ \ldots \ \mathbf{x}^{M-1}] \end{aligned} \quad (26)$$

SVD of $\mathbf{H}_{c0}$ yields

$$\begin{aligned} \mathbf{H}_{c0} &\equiv \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^T \\ &\simeq [\mathbf{U}_R \ \mathbf{U}_D] \begin{bmatrix} \boldsymbol{\Sigma}_R & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{V}_R^T \\ \mathbf{V}_D^T \end{bmatrix} \\ &= \mathbf{U}_R \boldsymbol{\Sigma}_R^{1/2} \boldsymbol{\Sigma}_R^{1/2} \mathbf{V}_R^T \end{aligned} \quad (27)$$

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23. The method of Claim 22, further obtaining fluid system matrices **(A, B, C, D)** approximately as follows:

$$
\begin{aligned}
\mathbf{D} &= \mathbf{Y}^0 && (28)\\
\mathbf{C} &\simeq \mathbf{U}_R \boldsymbol{\Sigma}_R^{1/2} && (29)\\
\mathbf{B} &\simeq \boldsymbol{\Sigma}_R^{-1/2} \mathbf{U}_R^T \mathbf{Y}^1 && (30)\\
\mathbf{A} &\simeq \boldsymbol{\Sigma}_R^{-1/2} \mathbf{U}_R^T \mathbf{H}_{c1} \mathbf{V}_R \boldsymbol{\Sigma}_R^{-1/2} && (31)
\end{aligned}
$$

24. The method of Claim 22, further comprising defining augmented $\mathbf{H}_{c01}$ and $\mathbf{H}_{c11}$ matrices as follows:

$$
\begin{aligned}
\mathbf{H}_{c01} &\equiv \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \\ \vdots \\ \mathbf{CA}^K \end{bmatrix} [\mathbf{x}^1 \ \mathbf{x}^2 \ \ldots \ \mathbf{x}^{M-1}] \\
&= \begin{bmatrix} \mathbf{y}_{c0}^1 & \mathbf{y}_{c0}^2 & \cdots & \mathbf{y}_{c0}^{M-1} \\ \mathbf{y}_{c1}^1 & \mathbf{y}_{c1}^2 & \cdots & \mathbf{y}_{c1}^{M-1} \\ \cdots & \cdots & \cdots & \cdots \\ \mathbf{y}_{cK}^1 & \mathbf{y}_{cK}^2 & \cdots & \mathbf{y}_{cK}^{M-1} \end{bmatrix} && (32)\\
\mathbf{H}_{c11} &\equiv \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \\ \vdots \\ \mathbf{CA}^K \end{bmatrix} \mathbf{A} [\mathbf{x}^1 \ \mathbf{x}^2 \ \ldots \ \mathbf{x}^{M-1}] \\
&= \begin{bmatrix} \mathbf{y}_{c1}^1 & \mathbf{y}_{c1}^2 & \cdots & \mathbf{y}_{c1}^{M-1} \\ \mathbf{y}_{c2}^1 & \mathbf{y}_{c2}^2 & \cdots & \mathbf{y}_{c2}^{M-1} \\ \cdots & \cdots & \cdots & \cdots \\ \mathbf{y}_{cK+1}^1 & \mathbf{y}_{cK+1}^2 & \cdots & \mathbf{y}_{cK+1}^{M-1} \end{bmatrix} && (33)
\end{aligned}
$$

where

$$
\begin{aligned}
\mathbf{y}_{ck}^n &\equiv \mathbf{CA}^k \mathbf{x}^n \\
&= \mathbf{y}^{n+k} - \sum_{i=1}^{N_I} \mathbf{y}_i^0 r_i^{n+k} - \sum_{i=1}^{N_I} \mathbf{y}_i^1 r_i^{n+k-1} - \\
&\quad \ldots - \sum_{i=1}^{N_I} \mathbf{y}_i^k r_i^n
\end{aligned}
$$

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25. The method of Claim 20, wherein at least some of the input signals are at least one of filtered through a low-pass filter, applied in multiple step inputs in a sequential manner, and applied in multiple pulse inputs in a sequential manner.

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